

ICS 691: Parallel Algorithms

Homework 4

Due: Dec 4, 2014

Instructions: You may discuss the problems with other students in the class, but you must write up the solutions on your own and list the names of the students with whom you discussed each problem.

1 Constructing Connected Components - 30 pts

In class we presented an algorithm for connected components which updates each node with the ID of the connected component it belongs to. Given the output of the algorithm, one can answer queries of the form "Do u and v belong to the same connected component?" in $O(1)$ time. Design an efficient parallel algorithm, that given the output of the connected components algorithm, lists the nodes of each connected components. The nodes of the same component should be listed contiguously. Analyze the resource requirements of your algorithm.

2 Convex hull intersections - 40 pts

Let U and L be a sequence of n points each of two convex polygonal chains. U and L are given in sorted order by increasing x-coordinates. Describe a parallel algorithm that finds where the two polygonal chains intersect. Analyze the time and work complexity of your algorithm and prove its correctness. (Hint: use approach mentioned in class)

3 Rankings in the plane-sweep tree (Exercise 6.24) - 30pts

Show how to construct the plane-sweep tree T of a set of n nonintersecting horizontal segments such that at each node v we also have the information about the rank of each endpoint stored at v in the list of covering segments stored at v . Your algorithm should run in $O(\log n)$ time using $O(n \log n)$ work. Do NOT use the optimal $O(\log \log n)$ merging algorithm.

4 BONUS:Half-plane intersections (Exercise 6.31) - 30 pts

Let P be a simple polygon with n vertices. The **kernel** of P , denoted by $K(P)$, is the set of points q in P such that, for any point z on the boundary of P , the segment zq lies entirely in P . (See Figure 6.21 in the textbook for an example of a polygon and its kernel).

- (a) (20 pts) Show that $K(P)$ is the intersection of n half-planes defined by the edges of P .
- (b) (10 pts) Deduce an $O(\log n)$ time algorithm to compute $K(P)$. What is the total number of operations used?